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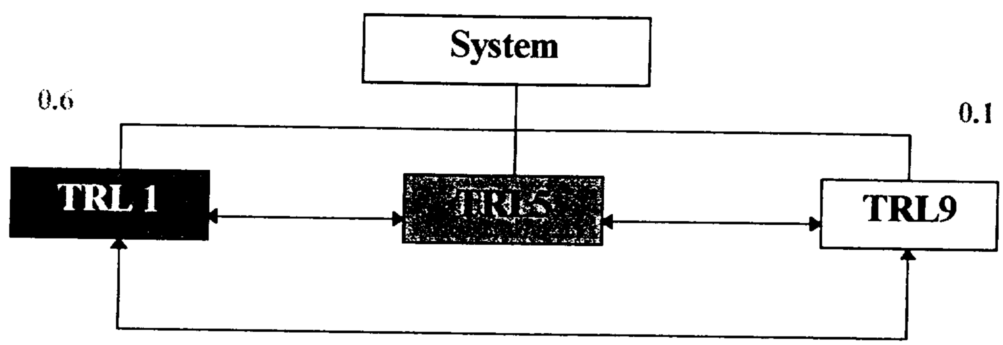
Systems Engineering Metrics:

Organizational Complexity and Product Quality Modeling

(H-28422D)

Complex Organizational Metric for Programmatic Risk Environments (COMPRÉ)



Final Report

For

NASA-MSFC

by

OR Applications

September 1997

Acknowledgments

The author wishes to thank Dr. Dale Thomas, EL01, for "architecting" the COMPRÉ program management tool, while serving as COTR for this effort. I also wish to thank Dr. Thomas for freely allowing me to utilize his technical concepts and systems complexity terminology. I wish to thank Mary Heck and Spencer Hill of SAIC for their exhaustive independent verification of the concepts described in this report, as well as for their cost and COMPRÉ data achieved while exercising the tool. Thanks are also due to Glen Ritter (EL21), Frank Fogle (EJ33), and Jim Parker (EL02) for three successive peer reviews of this concept.

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Complex Organizational Metric for Programmatic Risk Environments (COMPRÉ)

EXECUTIVE SUMMARY

Phase II of the development of a complexity measure for technologically risky programs is presented. This effort, taking place from April-October 1997, focuses on the utilization of a model, COMPRÉ, which quantifies this metric. The sensitivity of this metric to schedule duration, the determination of relevant programmatic benchmarks, and performance monitoring with COMPRÉ are also addressed.

Complexity Metric and Payoff

The proposed programmatic complexity metric, "normalized" risk, is the ratio of total programmatic risk to total programmatic return-on-investment. Total return-on-investment is the budget-weighted average of the payoffs for the individual subsystems of the program. These payoffs are functions of the degree of maturity for the various technologies of the subsystems of interest. The designations for these technological maturities are by NASA Technology Readiness Levels (TRL's). Payoffs are then quantitatively determined by nonlinear "basis" functions, which are mapped back to the TRL's.

Programmatic Risk

Total programmatic risk is determined by assessing the degree to which subsystems work with or against each other. Connectivity is determined by the programmatic architecture. Two subsystems which connect to each other, developmentally, each possess technologies which must be integrated. Risk is measured by the degree to which these technologies correlate, as well as the technological maturity. Furthermore, multiple organizations may be responsible for integrating subsystems, a significant risk factor. How the budget is allocated across these subsystems is also a major risk factor, since heavy investments in immature technologies, with potentially high returns, also carry significantly more risk than equivalent investments in more mature technologies. Finally, programmatic schedule

is a primary risk contributor. Longer schedules allow more time for maturing difficult technologies, but also increase the potential for showstoppers and unproductive inertia.

Major Conclusions

1. As a complexity metric, normalized risk significantly reduces the COMPRÉ model sensitivity to schedule duration.

2. A nonlinear regression between total programmatic cost and normalized risk proves highly significant statistically. Thus, normalized risk may prove to be a very effective benchmark for the complexity and cost of a given program - throughout its developmental stages.

3. A good benchmark for programs of excellence is a normalized risk of 1.0. Approximately 15% of all programs would be expected to have normalized risks less than 1.0.

4. Effective programmatic benchmarks may also be derived based on total program cost. For example, a fair benchmark for a $500M program would be a normalized risk of approximately 1.7.

5. The use of uncertainty analysis may be effective in providing quantitative clues to programmatic surprises.

1.0 INTRODUCTION

This report addresses the second phase of development for a complex organizational metric for programmatic risk environments, COMPRÉ. A number of issues concerning model utilization are provided in this report. These include the concept of "normalized" risk as a prescriptive measure for complex measures, the use of uncertainty analysis in predicting programmatic "surprises", and the use of principal components analysis to determine programmatic stability.

An issue concerning the sensitivity of COMPRÉ to schedule duration is also addressed by using normalized risk and marginal timeframe analysis. Also, the correlation between the normalized risk complexity measure and total programmatic cost is developed, leading to possible benchmarks for programs of various sizes. Finally, performance monitoring is addressed using innovative statistical quality control techniques.

2.0 STATISTICAL ACCELERATION OF COMPRÉ VALIDATION

This section includes all changes made to the model as the result of the IV&V effort. It also includes statistical calculations necessary for programmatic correlation to the model.

2.1 COMPRÉ BASIS FUNCTIONS

This section covers an analysis of the COMPRÉ basis functions described in the January 1997 report.

2.1.1 CONVERSION TO NASA TECHNOLOGY READINESS LEVELS

This conversion was performed to make COMPRÉ inputs consistent with NASA technology Readiness Levels (TRL's). The new equations are provided in Appendix A of this report.

2.1.2 SENSITIVITY PARAMETERS

This section presents an overview of the sensitivity of COMPRÉ results to "shape" and "scale" parameters in the basis functions. Figure 2.1.2-1 shows a representative sensitivity for the scale parameter of a system with TRL equal to 5. Note that the technological risk is increasing with decreasing TRL levels only between the values of 0.2 and 5. This result provides guidelines for logical values that such a scale parameter may take. Sensitivity analyses were performed for all shape and scale parameters for all basis functions. Additional details are provided in Appendix A.

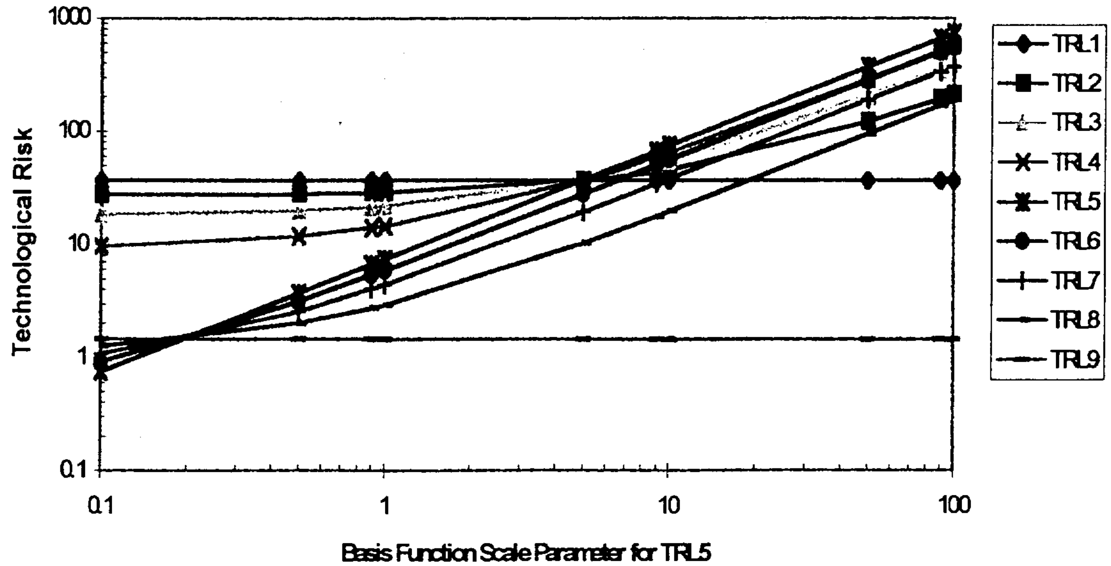

Figure 2.1.2-1. Sensitivity Analysis for TRL5 Basis Function Scale Parameter

2.2 COMPRÉ Uncertainty Analysis

This section provides an overview of uncertainty analyses using COMPRÉ. Additional details are provided in Appendix B of this report. Uncertainty analysis is used to determine the robustness, or lack thereof, of the risk and normalized risk metrics present in COMPRÉ for a given system.

A sample COMPRÉ risk analysis is shown in Figure 2.2-1 for the Hubble Space Telescope - Optical Telescope Assembly (HST-OTA). The variability of the OTA components is shown in this figure. The variability is defined as the degree to which the component contributes to the overall variance, or risk, of the OTA system. Percentages add up to greater than 100% due to combined interaction effects of the various components. That is, because two components interact with one another, the risk contribution of that interaction is attributed to both components. Note that the Optical Control Sub-Assembly and Structure/Mechanism have the highest contributions to overall risk, while the Optics and Fine Guidance components contribute the least.

Figure 2.2-2 shows the accompanying cost uncertainty analysis for the HST-OTA. Here the degree to which a component's cost may pose an unexpected surprise is measured by its relative variability. In other words, given a component's inherent variability, how much can it be expected to change, relative to itself, due to uncertainty in the projected cost of the component. The results show a near complete reversal of the risk analysis, with Fine Guidance, Optics, and Electronics leading the list of most likely surprises, from a cost standpoint. This result is in keeping with historical and logical program consequences, in which the subsystem considered least risky is often the one that ends up surprising the most, and vice versa.

Figure 2.2-3 provides the technology uncertainty analysis, with Fine Guidance, Electronics, and Optics being the most likely surprises from a technological standpoint. In case of point, Fine Guidance was the largest headache for the HST-OTA development.

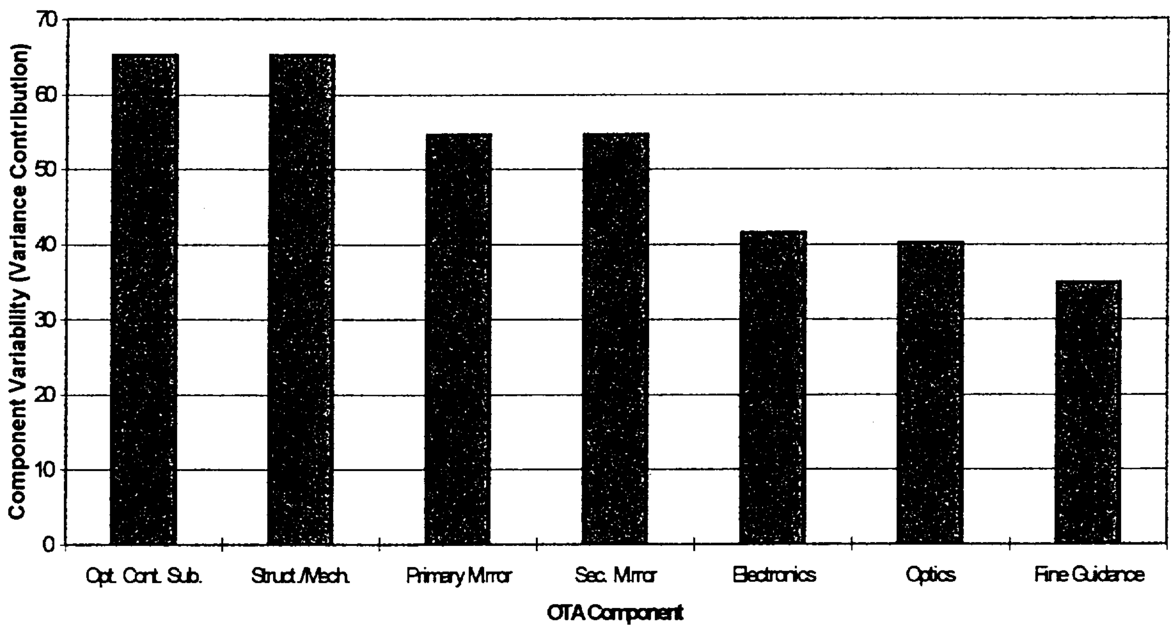

Figure 2. 2-1. Risk Analysis for HST-OTA

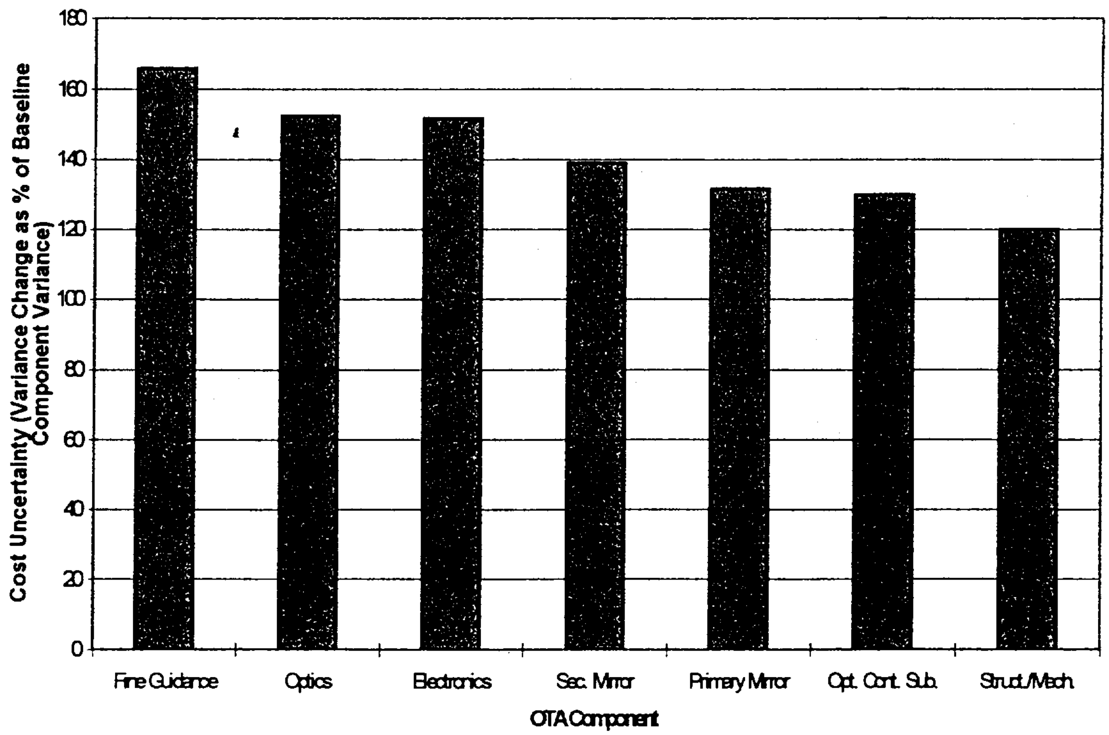

Figure 2. 2-2. Cost Uncertainty Analysis for HST-OTA

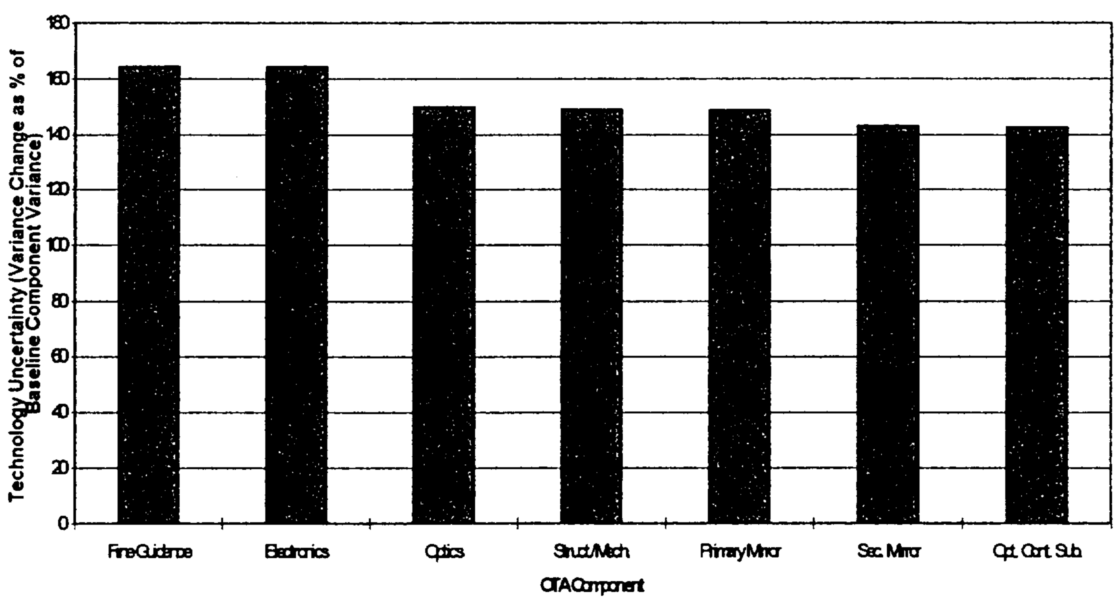

Figure 2. 2-3. Technology Uncertainty Analysis for HST-OTA

2.3 COMPRÉ Complexity Metric: Normalized Risk

Up to this point, we have dealt with the total programmatic risk, as quantified by COMPRÉ. Now, we introduce the concept of normalized risk. Normalized risk is defined as the ratio of total programmatic risk to total programmatic payoff. Certainly, we wish this metric to be as small as possible, since one would desire low risk and high payoff (to the degree it can be achieved) for any system developed. Thus, normalized risk represents the amount of programmatic risk accepted for each unit of programmatic reward. A close relative of normalized risk is the Sharpe Ratio of financial portfolio synthesis.

Figure 2.3-1 shows the relationship between schedule duration and normalized risk for the HST-OTA. This particular relationship is very typical. Note that the growth in normalized risk is roughly equal to the square root of the growth in schedule duration. Thus, normalized risk is very well bounded by schedule duration, and this sensitivity to schedule duration would not tend to be characterized as excessive.

Figure 2.3-2 shows how risk and normalized risk decrease over time as the HST-OTA schedule is expended. Note how slowly absolute risk degrades, while normalized risk decreases in a fairly linear fashion. The technical details of marginal timeframe analysis are provided in Appendix C.

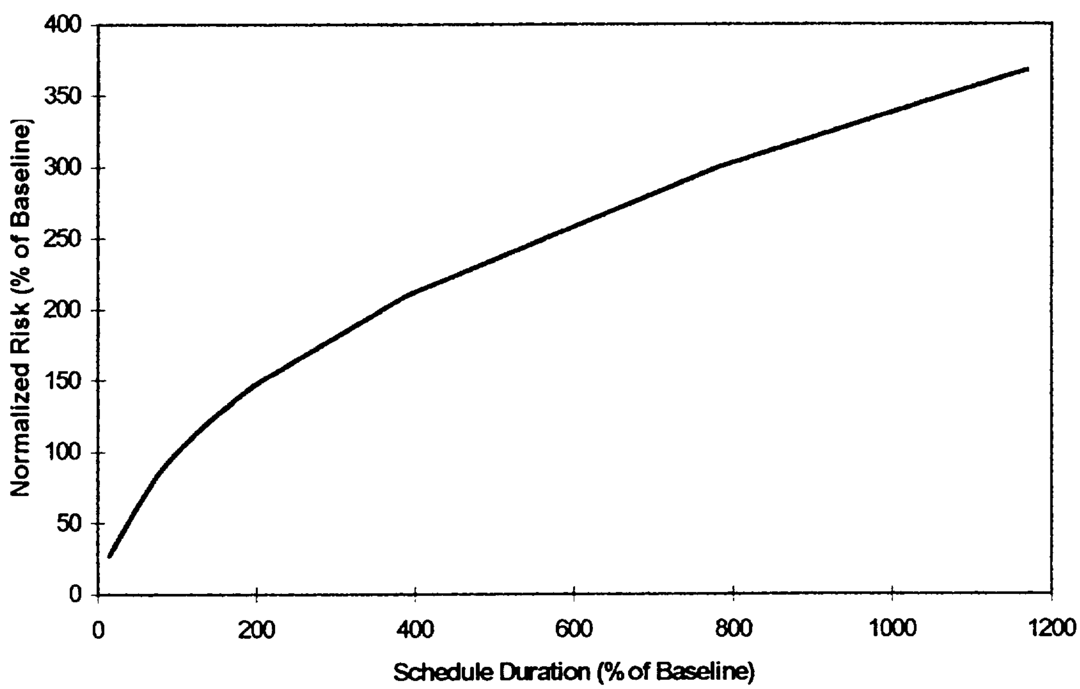

Figure 2.3-1. How Schedule Duration Affects Normalized Risk for HST-OTA

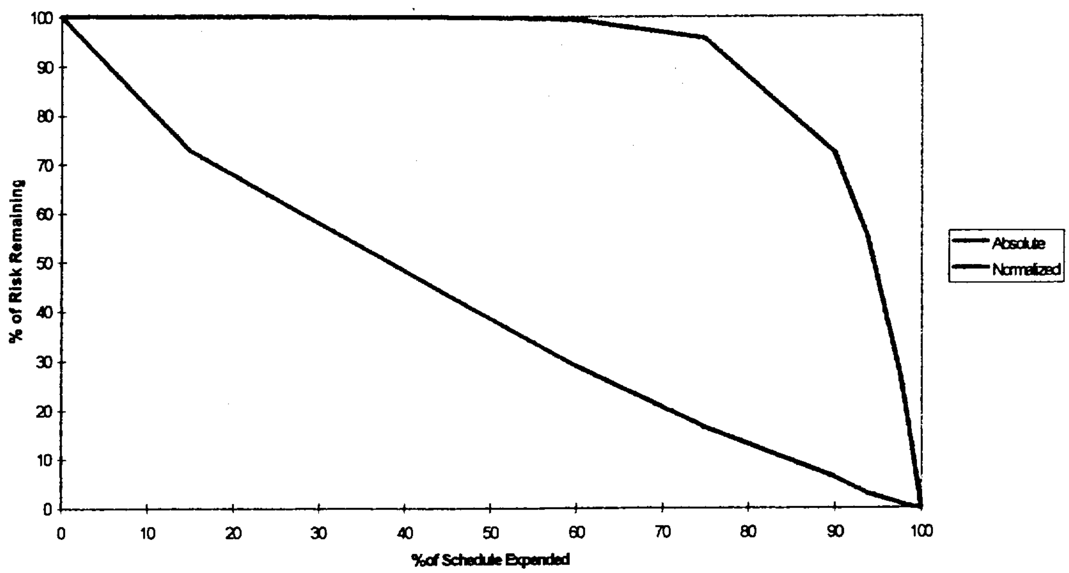

Figure 2.3-2. Risk and Normalized Risk as Schedule is Realized for HST-OTA

2.4 Relationship Between Normalized Risk and Total Programmatic Cost

COMPRÉ does not incorporate total programmatic cost directly. Budget allocations for subsystems and components are as a proportion of total programmatic cost, but that total cost is not used. The reason for this is that COMPRÉ must be flexible enough to measure system complexity independent of cost. If, in fact, complexity tends to increase with cost, COMPRÉ is not inherently aware of this. In fact, this reasoning presupposes that a large program could (but not necessarily will) be run as effectively as a small program. Conversely, a large, costly program need not be more complex that a small, cheap one, as far as COMPRÉ is concerned.

Therefore, it would be natural to test the correlation between COMPRÉ complexity, as measured by normalized risk, and total programmatic cost. In fact, this was done.

Figure 2.4-1 shows the total programmatic costs and COMPRÉ normalized risks (as provided by SAIC) for 18 developmental programs. The costs are provided in decreasing order, but notice the general tendency of normalized risk to decrease with cost, the major exception being HST. This graph leads one to consider the possible correlation between cost and normalized risk.

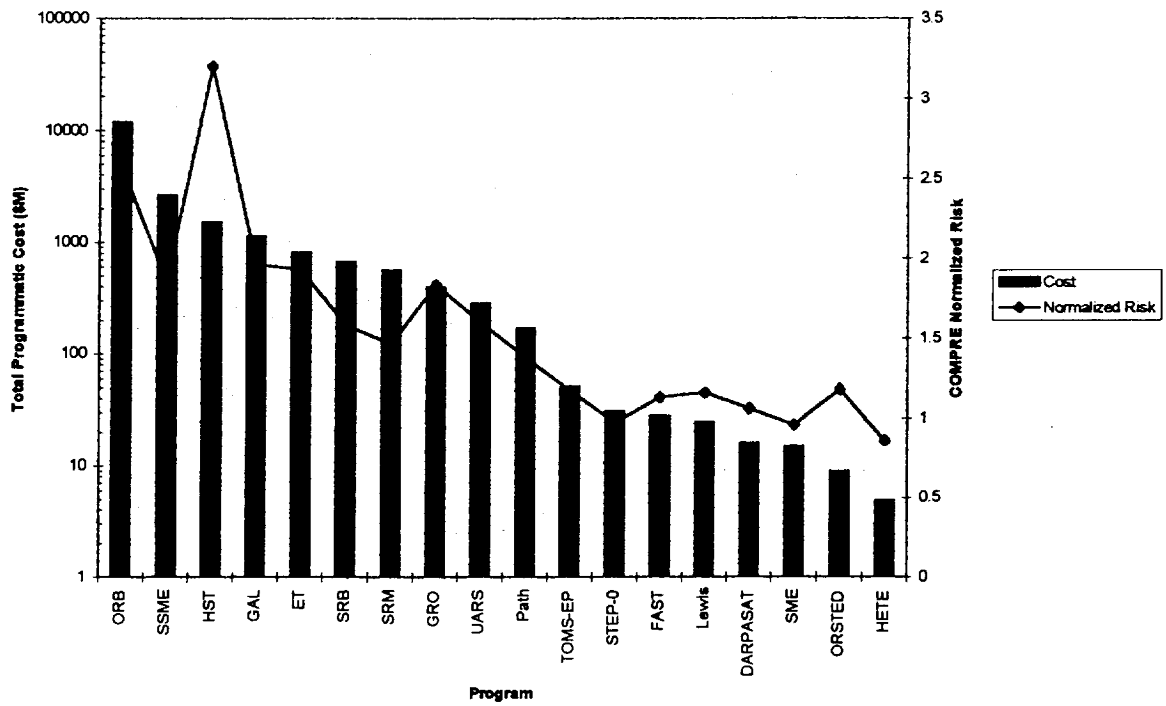

Figure 2.4-1. 18 Programs and Their Associated Normalized Risks and Costs

A nonlinear (geometric) regression between total programmatic cost and normalized risk is given as:

$$\frac{\sigma}{\lambda} = 0.6970C^{0.1451}$$

$$C = 12.0340(\frac{\sigma}{\lambda})^{6.8918}$$

$$R = 0.91$$

$$\alpha = 2.3E-07$$

where
σ = COMPRÉ risk (% standard deviation),
λ = COMPRÉ payoff,
C = total programmatic cost ($M 1996),
R = the correlation between normalized risk and total programmatic cost,
α = the probability of concluding the regression as significant statistically when it is not.

While the correlation of 0.91 indicates a good regression, it is the probability, 2.3E-07, which indicates that this correlation is not pure chance, given the number of data points and degrees of freedom. Figure 2.4-2 shows the actual and predicted values for normalized risk as a function of cost for the 18 programs. With the exception of HST, this is clearly an excellent fit, indicating that COMPRÉ has done a suitable job of capturing the system complexity in terms of normalized risk, as it relates to programmatic cost.

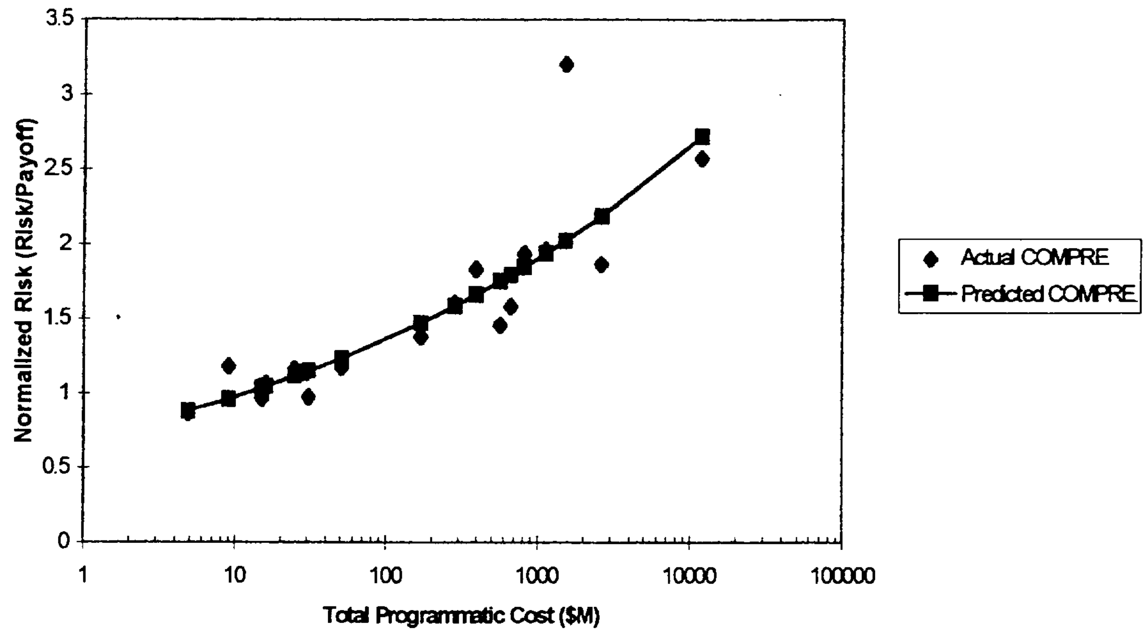

Figure 2.4-2. Actual and Predicted Normalized Risk as Function of Cost

3.0 STATISTICAL ACCELERATION OF COMPRÉ QUALITY MONITORING

This section provides the basis for monitoring the deviations from expectation and experience in a real-time, accelerated manner. It is the feedback process that simultaneously measures programmatic performance, while making recommendations for corrective action, when necessary. Technical details are provided in Appendix D.

3.1 COMPRÉ HYPOTHESIS TESTS

In order to monitor decision quality for recommendation purposes, measured conclusions must be capable of being drawn. To draw these conclusions, statistical hypothesis tests must be formulated. One such test was described in the Jan. 1997 Final Report. This test is determined by the null hypothesis, which is that the actual normalized risk for the program in question does not exceed the relevant normalized risk benchmark. The determination of such a benchmark is described in the next section.

3.2 BENCHMARKS AND TESTING PARAMETERS

Performance benchmarks and decision quality test parameters must be specified for the hypothesis tests. Test parameters include Type I and II error rates, which are the probabilities of drawing erroneous conclusions concerning program management performance relative to appropriate benchmarks.

Figure 3.2-1 shows the distribution of actual normalized risks for the 18 programs. Note that approximately 45% of the 18 programs have normalized risks below 1.25, while 90% are below 2.25. This shows the sensitivity displayed in this complexity metric, where small differences in normalized risk mean large differences in performance and cost.

Figure 3.2-2 shows the distribution of resampled normalized risks based on the 18 programs for 1000 resamples. A resample is a statistical re-simulation of all 18 programs, with the arithmetic mean serving as the resample value. Note that, unlike the histogram of actual normalized risks, approximately 85% of the 1000 resamples have normalized risks below 1.25, with roughly 15% below 1.0. Thus, a normalized risk of 1.0 might serve as an effective benchmark for excellence in programs.

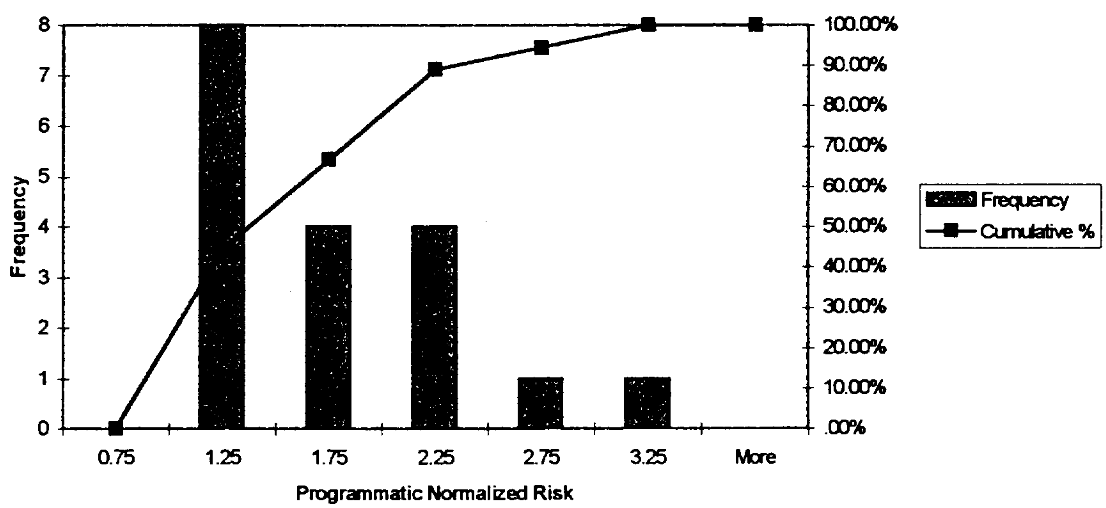

Figure 3.2-1. Actual Distribution of Normalized Risks for 18 Programs

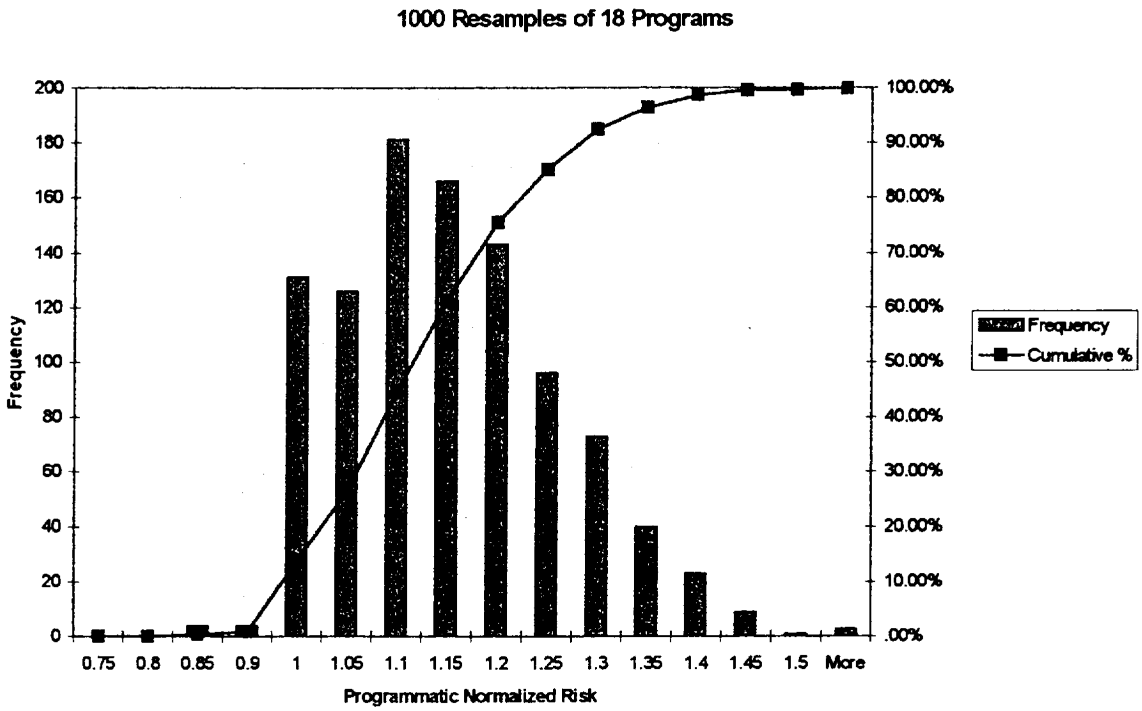

Figure 3.2-2. Resampled Distribution of Normalized Risks for 18 Programs

3.3 STATISTICAL ACCELERATORS

This section describes the application of statistical accelerators to maximize productive feedback on performance monitoring.

3.4 STATISTICAL CONCLUSION AND RECOMMENDATION FOR CORRECTIVE ACTION

Appendix D describes the technical details for achieving conclusiveness using programmatic benchmarks to perform performance monitoring.

Figure 3.4-1 provides an example of a quality decision being made. In this example, a rejection of the null hypothesis means that the program is underperforming its benchmark. That is, there is high confidence that the actual normalized risk is greater than the benchmark, which is undesirable.

Figure 3.4-2 shows the required number of COMPRÉ evaluations to draw a statistically significant conclusion for various values of the actual (unknown) normalized risk. Typically, 12-15 evaluations are required. This result may or may not be acceptable, depending on the frequency of data updates and the length of the program.

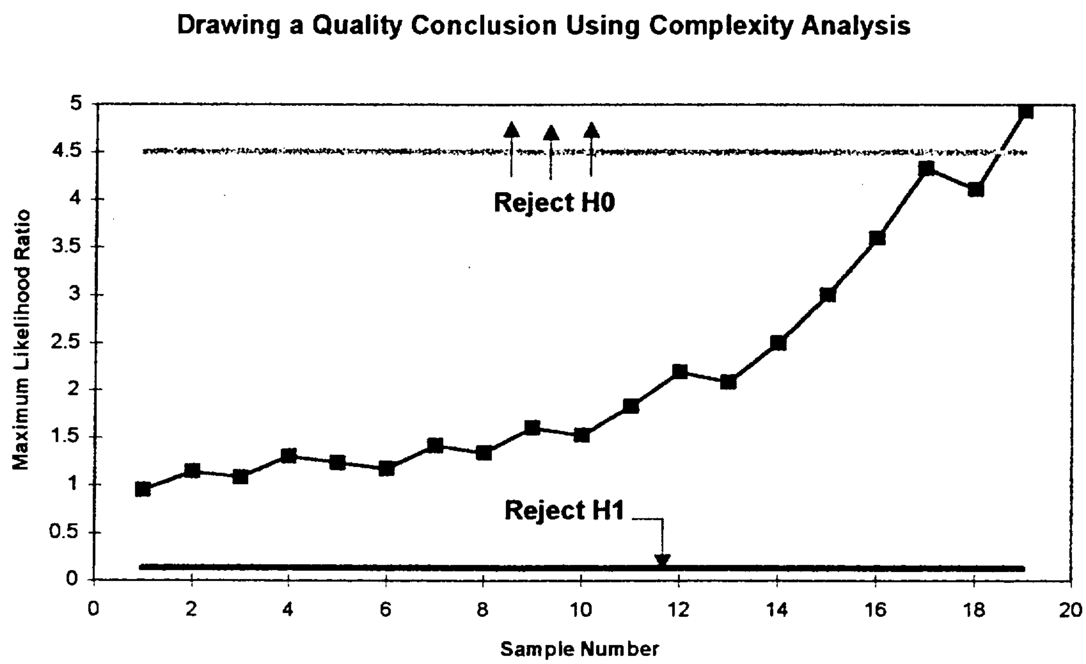

Figure 3.4-1. Determination of a Statistically Significant Conclusion

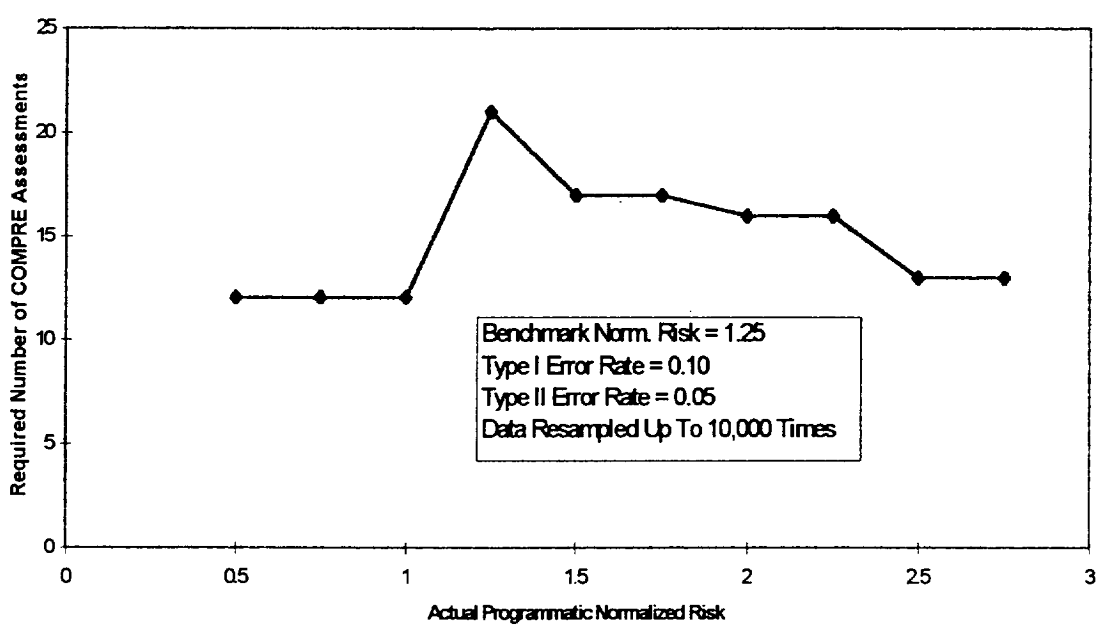

Figure 3.4-2. Required Number of COMPRÉ Evaluations to Draw a Statistically Significant Conclusion

4.0 CONCLUSIONS AND RECOMMENDATIONS

Conclusions

1. As a complexity metric, normalized risk significantly reduces the COMPRÉ model sensitivity to schedule duration.

2. A nonlinear regression between total programmatic cost and normalized risk proves highly significant statistically. Thus, normalized risk may prove to be a very effective benchmark for the complexity and cost of a given program - throughout its developmental stages.

3. A good benchmark for programs of excellence is a normalized risk of 1.0. Approximately 15% of all programs would be expected to have normalized risks less than 1.0.

4. Effective programmatic benchmarks may also be derived based on total program cost. For example, a fair benchmark for a $500M program would be a normalized risk of approximately 1.7.

5. The use of uncertainty analysis may be effective in providing quantitative clues to programmatic surprises.

Recommendations

1. Use COMPRÉ normalized risk as a systemic complexity metric.

2. Set benchmark normalized risk values in the vicinity of 1.0, with possible allowance for program size.

3. Use relative uncertainty analysis for the prediction of programmatic surprises.

4. Perform programmatic performance monitoring using statistical acceleration of COMPRÉ normalized risk.

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APPENDIX A: COMPRÉ Basis Functions

The general formulas for evaluating the technological maturities and programmatic risk are provided in Section 4.0 of the January 1997 report, with the overall methodology provided in Section 5.0 of the same report. This appendix provides the specific basis function manipulations to achieve system (and subsystem) level results.

The nine basis functions, corresponding to the nine NASA Technological Readiness Levels (TRL's), are given by:

$$
\begin{aligned}
r_1(t) &= e^t \\
r_2(t) &= \frac{t^2}{4} + \frac{3e^t}{4} \\
r_3(t) &= \frac{t^2}{2} + \frac{e^t}{2} \\
r_4(t) &= \frac{3t^2}{4} + \frac{e^t}{4} \\
r_5(t) &= t^2 \\
r_6(t) &= \frac{t}{4} + \frac{3t^2}{4} \\
r_7(t) &= \frac{t}{2} + \frac{t^2}{2} \\
r_8(t) &= \frac{3t}{4} + \frac{t^2}{4} \\
r_9(t) &= t
\end{aligned}
$$

Note that the basis functions for TRL's of 2, 3, 4, 6, 7, and 8 are linear combinations of the basis functions for TRL's of 1, 5, and 9. The expected technological payoff for the standard basis functions may be found by integration to be:

$$
\begin{aligned}
E[r_1(t)] &= \frac{e^t - 1}{t} \\
E[r_5(t)] &= \frac{t^2}{3} \\
E[r_9(t)] &= \frac{t}{2}
\end{aligned}
$$

The other basis function expectations may be calculated as linear functions of these expectations:

$$E[r_2(t)] = \frac{3E[r_1(t)]}{4} + \frac{E[r_5(t)]}{4}$$
$$E[r_3(t)] = \frac{E[r_1(t)]}{2} + \frac{E[r_5(t)]}{2}$$
$$E[r_4(t)] = \frac{E[r_1(t)]}{4} + \frac{3E[r_5(t)]}{4}$$
$$E[r_6(t)] = \frac{3E[r_5(t)]}{4} + \frac{E[r_9(t)]}{4}$$
$$E[r_7(t)] = \frac{E[r_5(t)]}{2} + \frac{E[r_9(t)]}{2}$$
$$E[r_8(t)] = \frac{E[r_5(t)]}{4} + \frac{3E[r_9(t)]}{4}$$

The standard basis covariances amongst themselves may be found through integration as:

$$C_{11}(t) = e^{2t}(\frac{1}{2t} - \frac{1}{t^2}) - \frac{1}{2t} + \frac{2e^t - 1}{t^2}$$
$$C_{55}(t) = \frac{4t^4}{45}$$
$$C_{99}(t) = \frac{t^2}{12}$$
$$C_{15}(t) = e^t(\frac{2t}{3} - 2 + \frac{2}{t}) + \frac{t}{3} - \frac{2}{t}$$
$$C_{19}(t) = e^t(\frac{1}{2} - \frac{1}{t}) + \frac{1}{2} + \frac{1}{t}$$
$$C_{59}(t) = \frac{t^3}{12}$$

Finally, covariances amongst the other (nonstandard) basis functions are evaluated as linear combinations of the standard basis covariances, as given below.

$$C_{12}(t) = \frac{3C_{11}(t)}{4} + \frac{C_{15}(t)}{4}$$
$$C_{13}(t) = \frac{C_{11}(t)}{2} + \frac{C_{15}(t)}{2}$$
$$C_{14}(t) = \frac{C_{11}(t)}{4} + \frac{3C_{15}(t)}{4}$$
$$C_{16}(t) = \frac{3C_{15}(t)}{4} + \frac{C_{19}(t)}{4}$$
$$C_{17}(t) = \frac{C_{15}(t)}{2} + \frac{C_{19}(t)}{2}$$
$$C_{18}(t) = \frac{C_{15}(t)}{4} + \frac{3C_{19}(t)}{4}$$

$$C_{22}(t) = \frac{9C_{11}(t)}{16} + \frac{6C_{15}(t)}{16} + \frac{C_{55}(t)}{16}$$
$$C_{23}(t) = \frac{3C_{11}(t)}{8} + \frac{4C_{15}(t)}{8} + \frac{C_{55}(t)}{8}$$
$$C_{24}(t) = \frac{3C_{11}(t)}{16} + \frac{10C_{15}(t)}{16} + \frac{3C_{55}(t)}{16}$$
$$C_{25}(t) = \frac{3C_{15}(t)}{4} + \frac{C_{55}(t)}{4}$$
$$C_{26}(t) = \frac{9C_{15}(t)}{16} + \frac{3C_{19}(t)}{16} + \frac{3C_{55}(t)}{16} + \frac{C_{59}(t)}{16}$$
$$C_{27}(t) = \frac{3C_{15}(t)}{8} + \frac{3C_{19}(t)}{8} + \frac{C_{55}(t)}{8} + \frac{C_{59}(t)}{8}$$
$$C_{28}(t) = \frac{3C_{15}(t)}{16} + \frac{9C_{19}(t)}{16} + \frac{C_{55}(t)}{16} + \frac{3C_{59}(t)}{16}$$
$$C_{29}(t) = \frac{3C_{19}(t)}{4} + \frac{C_{59}(t)}{4}$$

$$C_{33}(t)=\frac{C_{11}(t)}{4}+\frac{2C_{15}(t)}{4}+\frac{C_{55}(t)}{4}$$
$$C_{34}(t)=\frac{C_{11}(t)}{8}+\frac{4C_{15}(t)}{8}+\frac{3C_{55}(t)}{8}$$
$$C_{35}(t)=\frac{C_{15}(t)}{2}+\frac{C_{55}(t)}{2}$$
$$C_{36}(t)=\frac{3C_{15}(t)}{8}+\frac{C_{19}(t)}{8}+\frac{3C_{55}(t)}{8}+\frac{C_{59}(t)}{8}$$
$$C_{37}(t)=\frac{C_{15}(t)}{4}+\frac{C_{19}(t)}{4}+\frac{C_{55}(t)}{4}+\frac{C_{59}(t)}{4}$$
$$C_{38}(t)=\frac{C_{15}(t)}{8}+\frac{3C_{19}(t)}{8}+\frac{C_{55}(t)}{8}+\frac{3C_{59}(t)}{8}$$
$$C_{39}(t)=\frac{C_{19}(t)}{2}+\frac{C_{59}(t)}{2}$$

$$C_{44}(t)=\frac{C_{11}(t)}{16}+\frac{6C_{15}(t)}{16}+\frac{9C_{55}(t)}{16}$$
$$C_{45}(t)=\frac{C_{15}(t)}{4}+\frac{3C_{55}(t)}{4}$$
$$C_{46}(t)=\frac{3C_{15}(t)}{16}+\frac{C_{19}(t)}{16}+\frac{9C_{55}(t)}{16}+\frac{3C_{59}(t)}{16}$$
$$C_{47}(t)=\frac{C_{15}(t)}{8}+\frac{C_{19}(t)}{8}+\frac{3C_{55}(t)}{8}+\frac{3C_{59}(t)}{8}$$
$$C_{48}(t)=\frac{C_{15}(t)}{16}+\frac{3C_{19}(t)}{16}+\frac{3C_{55}(t)}{16}+\frac{9C_{59}(t)}{16}$$
$$C_{49}(t)=\frac{C_{19}(t)}{4}+\frac{3C_{59}(t)}{4}$$

$$C_{56}(t)=\frac{3C_{55}(t)}{4}+\frac{C_{59}(t)}{4}$$
$$C_{57}(t)=\frac{C_{55}(t)}{2}+\frac{C_{59}(t)}{2}$$
$$C_{58}(t)=\frac{C_{55}(t)}{4}+\frac{3C_{59}(t)}{4}$$

$$C_{66}(t) = \frac{9C_{55}(t)}{16} + \frac{6C_{59}(t)}{16} + \frac{C_{99}(t)}{16}$$
$$C_{67}(t) = \frac{3C_{55}(t)}{8} + \frac{4C_{59}(t)}{8} + \frac{C_{99}(t)}{8}$$
$$C_{68}(t) = \frac{3C_{55}(t)}{16} + \frac{10C_{59}(t)}{16} + \frac{3C_{99}(t)}{16}$$
$$C_{69}(t) = \frac{3C_{59}(t)}{4} + \frac{C_{99}(t)}{4}$$

$$C_{77}(t) = \frac{C_{55}(t)}{4} + \frac{2C_{59}(t)}{4} + \frac{C_{99}(t)}{4}$$
$$C_{78}(t) = \frac{C_{55}(t)}{8} + \frac{4C_{59}(t)}{8} + \frac{3C_{99}(t)}{8}$$
$$C_{79}(t) = \frac{C_{59}(t)}{2} + \frac{C_{99}(t)}{2}$$

$$C_{88}(t) = \frac{C_{55}(t)}{16} + \frac{6C_{59}(t)}{16} + \frac{9C_{99}(t)}{16}$$
$$C_{89}(t) = \frac{C_{59}(t)}{4} + \frac{3C_{99}(t)}{4}$$

Note that the limiting values for all covariances as t approaches 0 are all equal to 0. In some cases, L'Hôpital's Rule is employed to verify this.

Generally Weighted Basis Functions with Shape and Scale Parameters

The nine basis functions, corresponding to the nine NASA Technological Readiness Levels (TRL's), are given by:

$$r_1(t) = \alpha_1 e^{\beta_1 t}$$
$$r_2(t) = w_{21} r_1(t) + w_{25} r_5(t), w_{21} + w_{25} = 1$$
$$r_3(t) = w_{31} r_1(t) + w_{35} r_5(t), w_{31} + w_{35} = 1$$
$$r_4(t) = w_{41} r_1(t) + w_{45} r_5(t), w_{41} + w_{45} = 1$$
$$r_5(t) = \alpha_5 t^2$$
$$r_6(t) = w_{65} r_5(t) + w_{69} r_9(t), w_{65} + w_{69} = 1$$
$$r_7(t) = w_{75} r_5(t) + w_{79} r_9(t), w_{75} + w_{79} = 1$$
$$r_8(t) = w_{85} r_5(t) + w_{89} r_9(t), w_{85} + w_{89} = 1$$
$$r_9(t) = \alpha_9 t$$

Note that the basis functions for TRL's of 2, 3, 4, 6, 7, and 8 are combinations of the basis functions for TRL's of 1, 5, and 9. The expected technological payoff for the standard basis functions may be found by integration to be:

$$E[r_1(t)] = \frac{\alpha_1(e^{\beta_1 t} - 1)}{\beta_1 t}$$
$$E[r_5(t)] = \frac{\alpha_5 t^2}{3}$$
$$E[r_9(t)] = \frac{\alpha_9 t}{2}$$

The other basis function expectations may be calculated as linear functions of these expectations:

$$E[r_2(t)] = w_{21}E[r_1(t)] + w_{25}E[r_5(t)]$$
$$\vdots$$
$$E[r_8(t)] = w_{85}E[r_5(t)] + w_{89}E[r_9(t)]$$

The standard basis covariances amongst themselves may be found through integration as:

$$C_{11}(t) = \frac{\alpha_1^2}{\beta_1^2}\left(\frac{2e^{\beta_1 t} - e^{2\beta_1 t} - 1}{t^2}\right) + \frac{\alpha_1^2}{2\beta_1}\left(\frac{e^{2\beta_1 t} - 1}{t}\right)$$
$$C_{55}(t) = \frac{4\alpha_5^2 t^4}{45}$$
$$C_{99}(t) = \frac{\alpha_9^2 t^2}{12}$$
$$C_{15}(t) = \frac{2\alpha_1\alpha_5}{3}\left\{\left[\frac{t}{\beta_1} - 3\left(\frac{\beta_1 t - 1}{\beta_1^3 t}\right)\right]e^{\beta_1 t} + \frac{t}{2\beta_1} - \frac{3}{\beta_1^3 t}\right\}$$
$$C_{19}(t) = \frac{\alpha_1\alpha_9}{2}\left[\left(\frac{\beta_1 t - 2}{\beta_1^2 t}\right)e^{\beta_2 t} + \frac{1}{\beta_1} + \frac{2}{\beta_1^2 t}\right]$$
$$C_{59}(t) = \frac{\alpha_5\alpha_9 t^3}{12}$$

Finally, covariances amongst the other (nonstandard) basis functions are evaluated as combinations of the standard basis covariances, as given below.

$$C_{12}(t) = w_{21}C_{11}(t) + w_{25}C_{15}(t)$$
$$\vdots$$
$$C_{18}(t) = w_{85}C_{15}(t) + w_{89}C_{19}(t)$$

$$C_{22}(t) = w_{21}^2 C_{11}(t) + 2w_{21}w_{25}C_{15}(t) + w_{25}^2 C_{55}(t)$$

$$\vdots$$

$$C_{29}(t) = w_{21}C_{19}(t) + w_{25}C_{59}(t)$$

$$\vdots$$

$$C_{88}(t) = w_{85}^2 C_{55}(t) + 2w_{85}w_{89}C_{59}(t) + w_{89}^2 C_{99}(t)$$

Note that the limiting values for all covariances as t approaches 0 are all equal to 0. In some cases, L'Hôpital's Rule is employed to verify this.

APPENDIX B: Uncertainty Analysis

The following equations provide the means of conducting uncertainty analyses with respect to technology, cost, schedule, architecture, and organization.

Uncertainty Analysis for TRL Specification:

$$\frac{\partial\sigma^2}{\partial TRL_i} = \sum_{i=1}^{n} w_i(t) \sum_{j=1}^{n} w_j(t) \frac{\partial C_{ij}(t)}{\partial TRL_i}$$

$$\frac{\partial C_{ij}(t)}{\partial TRL_i} = \frac{1}{t}\int_0^t \{[\frac{\partial r_i(t)}{\partial TRL_i} - \frac{\partial E[r_i(t)]}{\partial TRL_i}][r_j(t) - E[r_j(t)]]\}dt$$

Uncertainty Analysis for Schedule Specification:

$$\frac{\partial\sigma^2}{\partial t} = \sum_{i=1}^{n} w_i(t) \sum_{j=1}^{n} w_j(t) \frac{\partial C_{ij}(t)}{\partial t}$$

$$\frac{\partial C_{ij}(t)}{\partial t} = \frac{1}{t}\{[r_i(t) - E[r_i(t)]][r_j(t) - E[r_j(t)]] - C_{ij}(t)\}$$

Uncertainty Analysis for Cost Specification:

$$\frac{\partial\sigma^2}{\partial w_k(t)} = \frac{1}{2} \sum_{i=1, i\neq k}^{n} w_i(t) \sum_{j=1, j\neq k}^{n} w_j(t) C_{ij}(t) + \sum_{j=1}^{n} w_j(t) C_{kj}(t)$$

Uncertainty Analysis for Organization Specification:

$$\sigma^2 = \sum_{i=1}^{n} w_i(t) \sum_{j=1}^{n} w_j(t) O_{ij}(t) C_{ij}(t)$$

$$\frac{\partial\sigma^2}{\partial O_{km}(t)} = w_k(t) w_m(t) C_{km}(t)$$

Uncertainty Analysis for Architecture:

$$\frac{\partial\sigma^2}{\partial A} = \sum_{i=1}^{n} w_i(t) \sum_{j=1}^{n} w_j(t) \frac{\partial C_{ij}(t)}{\partial A}$$

$$\frac{\partial C_{ij}(t)}{\partial A} = C_{ij}(t)$$

APPENDIX C: Marginal Timeframe Analysis

The following equations provide the process of performing marginal timeframe analysis over a partial period of the schedule duration.

$$\sigma^2(t_1,t_2) = \sum_{i=1}^{n} w_i(t) \sum_{j=1}^{n} w_j(t) O_{ij}(t) C_{ij}(t_1,t_2)$$

$$C_{ij}(t_1,t_2) = \frac{\int_{t_1}^{t_2} \{[r_i(t) - E[r_i(t)][r_j(t) - E[r_j(t)]]\} dt}{\int_{t_1}^{t_2} dt}$$

$$C_{ij}(t_1,t_2) = \frac{t_2}{t_2 - t_1} C_{ij}(t_2) - \frac{t_1}{t_2 - t_1} C_{ij}(t_1)$$

APPENDIX D: Programmatic Benchmarking

The following equations provide the null and alternate hypotheses in terms of the normalized risk of the system and benchmarks, respectively. Also provided is the mechanism for determining the Type I and II error rates.

$$\widetilde{R} = \frac{\sigma}{\lambda}$$

$$H_0 : \widetilde{R}_S \le \widetilde{R}_B$$

$$H_1 : \widetilde{R}_S > \widetilde{R}_B$$

$$\alpha = P(\widetilde{R}_S > \widetilde{R}_B | \widetilde{R}_S \le \widetilde{R}_B)$$

$$\beta = P(\widetilde{R}_S \le \widetilde{R}_B | \widetilde{R}_S > \widetilde{R}_B)$$

$$p = 1 - e^{-\frac{\widetilde{R}}{k}}$$

$$\hat{\alpha} = P\{\frac{\hat{p}_* - \hat{p}}{\hat{\sigma}_*} > \frac{\hat{p} - p_0}{\hat{\sigma}}\}$$

$$\hat{\beta} = P\{\frac{\hat{p}_* - \hat{p}}{\hat{\sigma}_*} \le \frac{\hat{p} - p_0}{\hat{\sigma}}\}$$

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The statistical refinement of a Complex Organizational Metric for Programmatic Risk Environments (COMPRÉ) suitable for NASA-MSFC's Systems Analysis and Integration Laboratory (SAIL) missions and objectives is presented. Particular attention is paid to model utilization, sensitivity to programmatic duration, benchmarking, and performance monitoring. Results indicate a powerful capacity for COMPRÉ normalized risk in predicting systemic complexity through total programmatic cost. Furthermore, appropriate benchmark statistics have been defined for programs of various sizes. Finally, performance monitoring may be effected in using COMPRÉ normalized risk.

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